Problems on Waveguides:

1. A rectangular waveguide is 5.1 cm by 2.4 cm (inside measurements). (a) Calculate the cutoff frequency of the dominant mode. (b) Calculate the lowest frequency and determine the mode closest to the dominant mode

$$f_c = 1.5x10^8 \sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2} = 1.5x10^8 \sqrt{(\frac{1}{0.051})^2 + (\frac{0}{0.024})^2} = 2.94\,GHz$$

(b) TM modes with m = 0 or n = 0 are not possible in a rectangular waveguide. $TE_{0,1}$, $TE_{2,0}$ and $TE_{0,2}$ modes are possible. For the modes, cutoff frequencies are as follows:

$$TE_{0,1}: f_c = 1.5x10^8 \sqrt{(\frac{0}{0.051})^2 + (\frac{1}{0.024})^2} = 6.25\,GHz$$

$$TE_{2,0}: f_c = 1.5x10^8 \sqrt{(\frac{2}{0.051})^2 + (\frac{0}{0.024})^2} = 5.882\,GHz$$

$$TE_{0,2}: f_c = 1.5x10^8 \sqrt{(\frac{0}{0.051})^2 + (\frac{2}{0.024})^2} = 12.5\,GHz$$

Hence, the $TE_{2,0}$ mode has the lowest cutoff frequency of all modes except the $TE_{1,0}$ dominant mode.

2. A wave is propagated in a parallel-plane waveguide. The frequency is 6 GHz, and the plane separation is 3 cm. Calculate: (a) the cutoff wavelength for the dominant mode; (b) wavelength in the waveguide; (c) the group and phase velocities; (d) characteristic wave impedance

(a) $\lambda_c = \frac{2a}{m} = \frac{2(3)}{1} = 6\,cm$

(b) $\lambda = \frac{c}{f} = \frac{3x10^8}{6x10^9} = 0.05\,m = 5\,cm$

(c) For the group velocity:

$$v_g = c\sqrt{1-(\lambda/\lambda_c)^2} = 3x10^8\sqrt{1-(0.05/0.06)^2} = 1.658\,x10^8\ m/s$$

For the phase velocity:

$$c^2 = v_g v_p;\quad v_p = c^2/v_g = (3x10^8)^2/1.658x10^8 = 5.428x10^8\ m/s$$

(d) $Z_{TE} = \frac{Z_O}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{120\pi}{\sqrt{1-(0.05/0.06)^2}} = 682\,\Omega$

3. It is necessary to propagate a 10-GHz signal in a waveguide whose wall separation is 6 cm. What is the greatest number of half-waves of electric intensity which it will be possible to establish between two walls (i.e., the largest value of m)? Calculate the

guide wavelength and the characteristic wave impedance for this mode of propagation.

Solution:

$$\lambda = \frac{c}{f} = \frac{3x10^8}{10x10^9} = 0.03\,m = 3\,cm$$

The wave will propagate in the waveguide as long as the waveguide's cutoff wavelength is greater than the free-space wavelength of the signal: $\lambda_c > \lambda$

When m = 1:

$$\lambda_c = \frac{2a}{m} = \frac{2(6)}{1} = 12\,cm$$ This mode will propagate.

When m = 2:

$$\lambda_c = \frac{2a}{m} = \frac{2(6)}{2} = 6\,cm$$ This mode will propagate.

When m = 3:

$$\lambda_c = \frac{2a}{m} = \frac{2(6)}{3} = 4\,cm$$ This mode will propagate.

When m = 4:

$$\lambda_c = \frac{2a}{m} = \frac{2(6)}{4} = 3\,cm$$ This mode will NOT propagate.

The greatest number of half-waves of electric intensity that can be established between the walls is 3.

For the guide wavelength:

$$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{3}{\sqrt{1-(3/4)^2}} = 4.54\,cm$$

For the characteristic wave impedance

$$Z_{TE} = \frac{Z_O}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{120\pi}{\sqrt{1-(0.03/0.04)^2}} = 570\,\Omega$$

4. Calculate the cutoff wavelength, the guide wavelength and the characteristic wave impedance of a circular waveguide whose internal diameter is 4 cm, for a 10-GHz signal propagated in it in the $TE_{1,1}$ mode.

$$\lambda = \frac{c}{f} = \frac{3x10^8}{10x10^9} = 0.03\,m = 3\,cm$$

$$\lambda_c = \frac{2\pi\, r}{B_{m,n}} = \frac{2\pi\,(4/2)}{1.84} = 6.82\,cm$$

$$Z_{TE} = \frac{Z_O}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{120\pi}{\sqrt{1-(0.03/0.0682)^2}} = 420\,\Omega$$