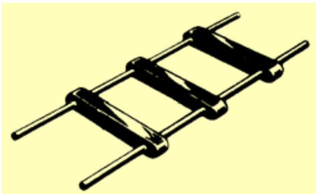
TRANSMISSION LINES

- A system of conductors having a precise geometry and arrangement that is used to transfer power from source to load with minimum loss.
- Means of conveying information from one point to another.
- The conductive connections between system elements which carry signal power.

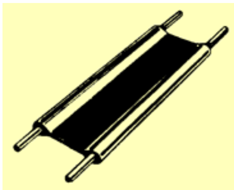
<u>Types of Transmission Lines</u>

A. DIFFERENTIAL OR BALANCED LINE – where neither conductor is grounded

1. **Two-Wire Open Lines** are parallel lines and have uses such as power lines, rural telephone lines, and telegraph lines. This type of line has high radiation losses and is subject to noise pickup.

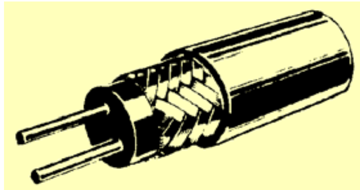


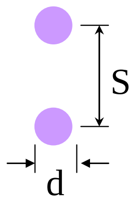
2. **Twin Lead** has parallel lines and is most often used to connect televisions to their antennas.



3. **A TWISTED PAIR** consists of two insulated wires twisted together. This line has high insulation loss.

4. **A SHIELDED PAIR** has parallel conductors separated by a solid dielectric and surrounded by copper braided tubing. The conductors are balanced to ground.

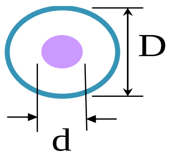




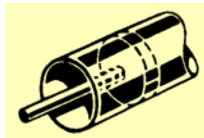
$$Z_o = 276 \log \frac{2S}{d}$$

B. SINGLE-ENDED OR UNBALANCED LINE – where one conductor is grounded

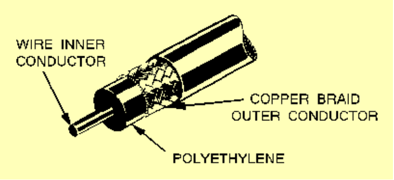
1. **RIGID COAXIAL LINE** contains two concentric conductors insulated from each other by spacers. Some rigid coaxial lines are pressurized with an inert gas to prevent moisture from entering. High frequency losses are less than with other lines.



$$Z_o = \frac{138}{\sqrt{\varepsilon_r}} \log \frac{D}{d}$$



2. **FLEXIBLE COAXIAL LINES** consist of a flexible inner conductor and a concentric outer conductor of metal braid. The two are separated by a continuous insulating material.



Velocity Factor and Dielectric Constant

Material	Velocity Factor (k)	Relative Dielectric Constant (ε_r)
Vacuum	1.0000	1.0000
Air	0.9997	1.0006
Teflon Foam	0.8200	1.4872
Teflon	0.6901	2.1000
Polyethylene	0.6637	2.2700
Paper. praffined	0.6325	2.5000
Polysterene	0.6325	2.5000
Polyvinyl chloride	0.5505	3.3000
Rubber	0.5774	3.0000
Mica	0.4472	5.0000
Glass	0.3651	7.5000

LOSSES IN A TRANSMISSION LINE

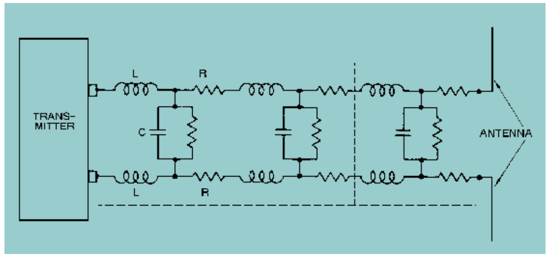
COPPER LOSSES can result from power (I^2R) loss, in the form of heat, or skin effect. These losses decrease the conductivity of a line.

DIELECTRIC LOSSES are caused by the heating of the dielectric material between conductors, taking power from the source.

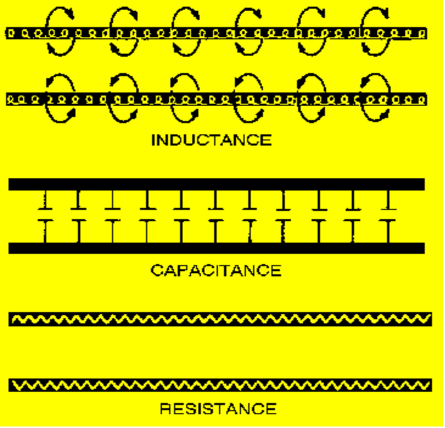
RADIATION AND INDUCTION LOSSES are caused by part of the electromagnetic fields of a conductor being dissipated into space or nearby objects.

A transmission line is either electrically **LONG** or **SHORT** if its physical length is not equal to 1/4λ for the frequency it is to carry.

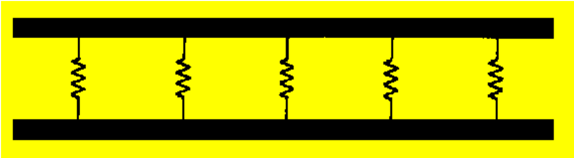
LUMPED CONSTANTS are theoretical properties (inductance, resistance, and capacitance) of a transmission line that are lumped into a single component.



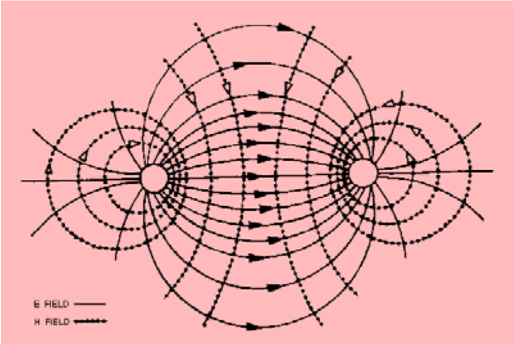
DISTRIBUTED CONSTANTS are constants of inductance, capacitance and resistance that are distributed along the transmission line.



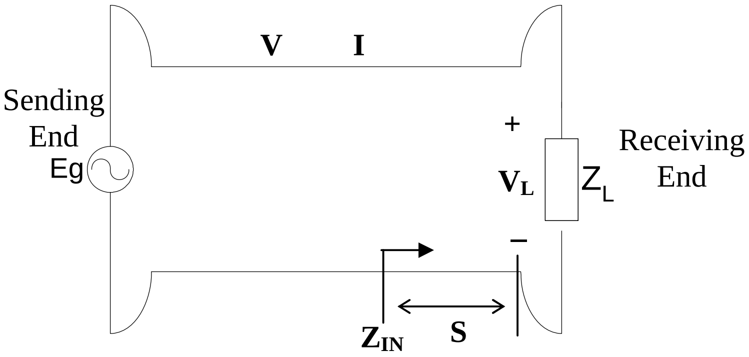
LEAKAGE CURRENT flows between the wires of a transmission line through the dielectric. The dielectric acts as a resistor.



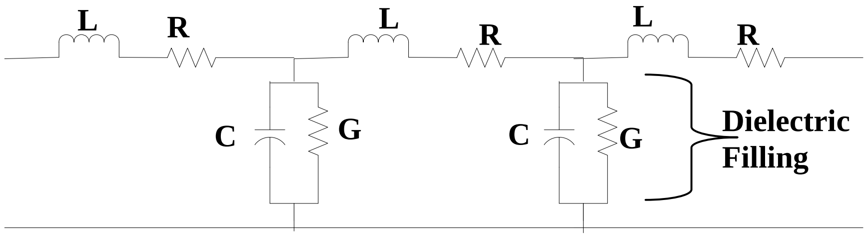
An **ELECTROMAGNETIC FIELD** exists along transmission line when current flows through it.



INTERACTIVE CIRCUIT



GENERAL EQUIVALENT CIRCUIT



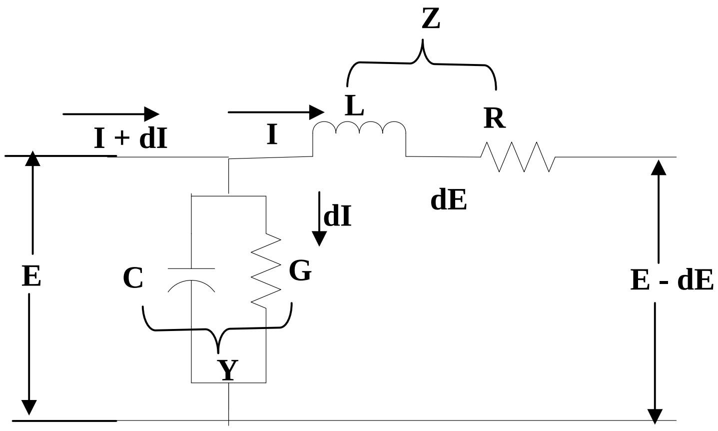
R, L, G & C are all per unit length

Note: At ***RF***, R and G are ignored or line is considered lossless

Where:	R = Ω / unit l	G = S / unit l
	L = H / unit l	C = F / unit l

CHARACTERISTIC IMPEDANCE (Zo)

- Reference input impedance
- Impedance measured at the input when its length is infinite
- Also known as the *surge impedance*



Section of TL

Where: $Z = R + j\omega L$, series impedance / section
$Y = G + j\omega C$, shunt impedance / section

By KCL:

$I + dI = I + EY$

$dI = EY\,(dS)$ --------- (1)

$dE = IZ\,(dS)$ ---------- (2)

$$\frac{dI}{dS} = EY \qquad \text{--------- (3)}$$

$$\frac{dE}{dS} = IZ \qquad \text{--------- (4)}$$

By KVL:

$E - dE = E - IZ$

Differentiate I and E with respect to S:

$$\frac{d^2I}{dS^2} = \frac{d^2I}{dS^2} = IZY$$

$$\frac{d^2E}{dS^2} = \frac{d^2E}{dS^2} = EYZ$$

General Solution :

$I = I_1 e^{\sqrt{ZY}} + I_2 e^{-\sqrt{ZY}}$ --------- (5)

$E = E_1 e^{\sqrt{ZY}} + E_2 e^{-\sqrt{ZY}}$ --------- (6)

But $\delta = \sqrt{ZY}$ = complex propagation constant

- **Propagation constant, δ, determines the variation of V or I with distance along the line: $V = V_s e^{-S\delta}$; $I = I_s e^{-S\delta}$, where V_S, and I_S are the voltage and current at the source end, and S = distance from source.**

  $\delta = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$

  where α = attenuation constant (neper/m or dB/m)
  β = phase delay constant (rad/m)

  substitute (6) to (4)

$$\frac{d(E_1 e^{\sqrt{ZY}} + E_2 e^{-\sqrt{ZY}})}{dS} = IZ$$

$$I = \frac{E_1 \sqrt{ZY}\, e^{\sqrt{ZY}} - E_2 \sqrt{ZY}\, e^{-\sqrt{ZY}})}{Z}$$

Compare this to (5)

$$I_1 = \frac{E_1 \sqrt{ZY}}{Z} = \frac{E_1}{\sqrt{Z/Y}}$$

therefore

$$\boxed{Z_o = \sqrt{\frac{Z}{Y}}} \qquad \boxed{Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

--------- (7)

where $\quad I_1 = E_1 / Zo \quad$ **and** $\quad I_2 = -E_2 / Zo$

At Radio Frequency

$$\boxed{Z_o = \sqrt{\frac{L}{C}}} \qquad \text{--------- (8)}$$

WAVELENGTH

- distance traveled by a point in the time required to complete one cycle.

$$\boxed{v = \frac{c}{\sqrt{\varepsilon_r}} = kc; \quad \lambda = \frac{v}{f}} \qquad \text{--------- (9)}$$

Where:

v = velocity of propagation along the line
f = frequency of operation
C = velocity of light , 3 x 10 ^{8} m/s
k = velocity factor, 0<k<1

STANDING WAVE

- **an interference pattern made by two sets of traveling waves going on opposite direction.**

From (6)

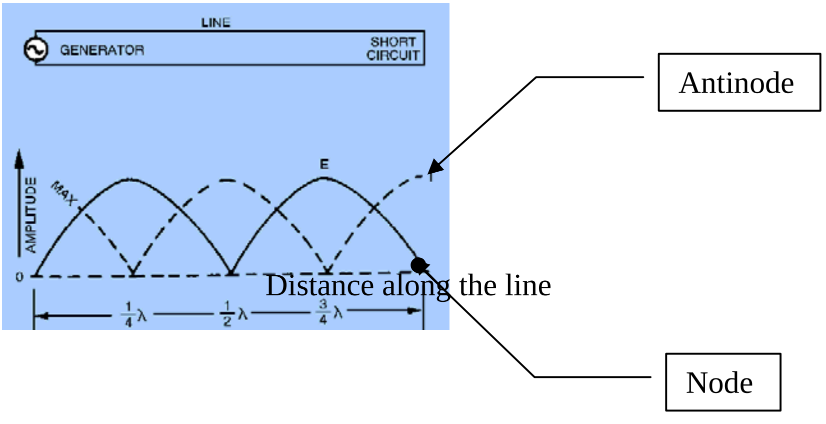
$$E = \underbrace{E_1 e^{\sqrt{ZY}\,s}}_{\text{Incident}} + \underbrace{E_2 e^{-\sqrt{ZY}\,s}}_{\text{Reflected}}$$

at the load; S = 0 , E = E_L

$$E_L = E_1 + E_2 \qquad \text{--------- (10)}$$

$$I_L = I_1 + I_2 \qquad \text{--------- (11)}$$

LOSSLESS LINE TERMINATED WITH SHORT CIRCUIT



$$\Gamma = \frac{-I_2}{I_1} = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma| \angle \phi$$

If $\Gamma = 0$, $Z_L = Z_o$ (perfect match)
$\Gamma = 1$, $Z_L = \infty$ (open circuit)
$\Gamma = -1$, $Z_L = 0$ (short circuit)

REFLECTION COEFFICIENT, Γ

- A measure of the degree of mismatch between the load and the line

$$\Gamma = \frac{E_2}{E_1} = \frac{-I_2 Z_O}{I_1 Z_O} = \frac{-I_2}{I_1} \qquad \text{--------- (12)}$$

From (10)

$E_L = E_1 + E_2$

$I_L Z_L = I_1 Z_o - I_2 Z_o$

$I_1 Z_L + I_2 Z_L = I_1 Z_{Lo} - I_2 Z_o$

$I_1 (Z_L - Z_o) = - I_2 (Z_L + Z_o)$

STANDING WAVE RATIO (SWR)

- ratio of maximum to minimum I or V

$$SWR = \frac{V_{MAX}}{V_{MIN}} = \frac{1+|\Gamma|}{1-|\Gamma|} \qquad \text{--------- (13)}$$

$$\Gamma = \frac{SWR - 1}{SWR + 1} \qquad \text{-------- (14)}$$

If Z_L is purely resistive

$$SWR = \frac{R_L}{Z_O} \quad \text{if } R_L > Z_O$$

$$SWR = \frac{Z_O}{RL} \quad \text{if } Z_O > R_L$$

VOLTAGE AND CURRENT AT ANY POINT ALONG THE LINE

From (10)

$$[\, E_L = E_1 + E_2 \,] \;/\; E_1$$

$$\frac{E_L}{E_1} = 1 + \Gamma \qquad E_1 = \frac{E_L}{1+\Gamma}$$

$$E_1 = \frac{E_L}{1 + \dfrac{(Z_L - Z_o)}{(Z_L + Z_o)}}$$

$$E_1 = \frac{E_L\ (Z_L + Z_o)}{2\,Z_L} \qquad \text{--------- (15)}$$

$$E_2 = \Gamma E_1 = \frac{E_L\ (Z_L + Z_o)\Gamma}{2\,Z_L} \qquad \text{-------- (16)}$$

Substitute to (6)

$$\boxed{E = \frac{E_L\left(Z_L + Z_O\right)}{2Z_L}\left(e^{\sqrt{ZY}S} + \Gamma e - ^{\sqrt{ZY}S}\right)}$$

--------- (17)

Equation of voltage at any point along the line

For current,

$$\boxed{I = \frac{I_L\left(Z_L + Z_O\right)}{2Z_L}\left(e^{\sqrt{ZY}S} - \Gamma e - ^{\sqrt{ZY}S}\right)}$$

--------- (18)

Equation of current at any point along the line

INPUT IMPEDANCE, Z_{IN}

$$Z_{IN} = \frac{E}{I} = \frac{\dfrac{E_L(Z_L + Z_o)}{2\,Z_L}\ \left(e^{\sqrt{ZY}\,S} + \Gamma\, e^{-\sqrt{ZY}\,S}\right)}{\dfrac{I_L(Z_L + Z_o)}{2\,Z_O}\ \left(e^{\sqrt{ZY}\,S} - \Gamma\, e^{-\sqrt{ZY}\,S}\right)}$$

If $\Gamma = \dfrac{(Z_L - Z_o)}{(Z_L + Z_o)}$ $,\delta = \sqrt{ZY}$

(substitute)

$$Z_{IN} = Zo \ \frac{e^{\delta S} + \left(\dfrac{(Z_L - Z_o)}{(Z_L + Z_o)}\right) e^{-\delta S}}{e^{\delta S} - \left(\dfrac{(Z_L - Z_o)}{(Z_L + Z_o)}\right) e^{-\delta S}}$$

$$Z_{IN} = Zo \ \frac{e^{\delta S}\ \ (Z_L + Zo) + (Z_L - Zo)\, e^{-\delta S}}{e^{\delta S}\ \ (Z_L + Zo) - (Z_L - Zo)\, e^{-\delta S}}$$

$$Z_{IN} = Z_o \frac{[Z_L (e^{\delta S} + e^{-\delta S}) + Zo (e^{\delta S} - e^{-\delta S})] \times 1/2}{[Z_O (e^{\delta S} + e^{-\delta S}) + Z_L (e^{\delta S} - e^{-\delta S})] \times 1/2}$$

Recall: $\sinh A = \dfrac{e^A - e^{-A}}{2}$

$\cosh A = \dfrac{e^A + e^{-A}}{2}$

$\tanh A = \dfrac{e^A - e^{-A}}{e^A + e^{-A}}$

$$Z_{IN} = Z_o \frac{[Z_L \cosh \delta S + Z_O \sinh \delta S](1/\cosh\delta S)}{[Z_O \cosh \delta S + Z_L \sinh \delta S](1/\cosh\delta S)}$$

$$\boxed{Z_{IN} = Z_O \frac{[Z_L + Z_O \tanh \delta S]}{[Z_O + Z_L \tanh \delta S]}} \text{ --------- (19)}$$

Manipulating tanh δS:

CASE I:

$\alpha \neq 0$; $\beta = 0$ $\delta = \alpha$

say $\alpha = 0.1; S = 2m; \delta = 0.2$

$\tanh(0.2) = 0.1974$

CASE II:

$\alpha \neq 0$; $\beta \neq 0$ $\delta S = \alpha S + j\beta S$

say $\alpha = 0.1; \beta = 0.2; S = 10m$; $\delta S = 1 + j2$

$$\tanh A = \frac{e^{(1+j2)} - e^{-(1+j2)}}{e^{(1+j2)} + e^{-(1+j2)}}$$

$e^{(1+j2)} = e^1 e^{j2}$

Recall Euler's Identity

$e^{jA} = 1\angle\pm A = \cos A \pm j\sin A$

CASE III:

$\alpha = 0$; $\beta \neq 0$ $\delta S = j\beta S$

$\tanh\delta S = \tanh j\beta S$

let $\beta S = A$

$$\tanh jA = \frac{e^{jA} - e^{-jA}}{e^{jA} + e^{-jA}}$$

$$\tanh jA = \frac{\cos A + j \sin A - (\cos A - j \sin A)}{\cos A + j \sin A + (\cos A - j \sin A)}$$

$$= \frac{2 j\sin A}{2 \cos A}$$

$\tanh jA = j \tan A$

therefore: tanh $j\beta S$ = tan $j\beta S$ ---- substitute in (19)

$$\boxed{Z_{IN} = Z_O \frac{[Z_L + jZ_O \tan \beta S]}{[Z_O + jZ_L \tan \beta S]}} \text{ --------- (20)}$$

- for lossless line

Note: cosh jx = cosx
sinhjx = jsinx

LOSSLESS TRANSMISSION LINE

1. No attenuation
2. No power loss (R=0, G=0)

Wavelength : distance that provides a phase shift 2π radian

$$\boxed{\lambda = \frac{v}{f} = \frac{360^o}{\beta} = \frac{2\pi}{\beta}}$$

$$\boxed{v = \frac{2\pi f}{\beta} = \frac{\omega}{\beta}} \text{ --------- (21)}$$

Complex Propagation Constant

$$\delta = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta \quad ; \quad j\beta = j\omega\sqrt{LC}$$

$$\boxed{\frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}}$$

$$\boxed{v = \frac{1}{\sqrt{LC}}} \text{ --------- (22)}$$

INPUT IMPEDANCE, Z_{IN}

$$Z_{IN} = Zo \frac{[Z_L + j Z_O \tan \beta S]}{[Z_O + j Z_L \tan \beta S]}$$

CASE I: $Z_L = Z_O$ (matched line)

$$Z_{IN} = Zo \cancelto{1}{\frac{[Z_L + j Z_O \tan \beta S]}{[Z_O + j Z_L \tan \beta S]}}$$

$$\boxed{Z_{IN} = Z_O}$$

CASE II: $Z_L = 0$ (short circuited line)

$$Z_{IN} = Zo \frac{j Z_O \tan \beta S}{Z_O}$$

$$\boxed{Z_{IN} = jZ_O \tan \beta S} \text{ --------- (23)}$$

CASE III: $Z_L = \alpha$ (open circuited line)

$$Z_{IN} = Zo \frac{[\alpha + j Z_O \tan \beta S]}{[Z_O + j \alpha \tan \beta S]} = \frac{\alpha}{\alpha}$$

By L'Hospital's Rule

$$Z_{IN} = Z_o \frac{[(Z_L/Z_L) + j(Z_O/Z_L)\tan\beta S]}{[(Z_O/Z_L) + j(Z_L/Z_L)\tan\beta S]}$$

1 0

$$Z_{IN} = Z_o \frac{1}{j\tan\beta S}$$

then

$$\boxed{Z_{IN} = -jZ_O \cot\beta S} \quad \text{--------- (24)}$$

$Z_{OC}Z_{SC} = j\,Z_O \tan\beta S\;(Z_O/\,j\tan\beta S) = Z_O^2$

$$\boxed{Z_O = \sqrt{Z_{OC}Z_{SC}}} \quad \text{--------- (25)}$$

Z_{IN} for special lengths

CASE I: $S = \lambda/4$ (quarter wavelength)

$$Z_{IN} = Z_o \frac{\{Z_L + j\,Z_O \tan[(2\pi/\lambda)(\lambda/4)\}}{\{Z_O + j\,Z_L \tan[(2\pi/\lambda)(\lambda/4)]\}} = \frac{\alpha}{\alpha}$$

since $\tan(\pi/2) = \alpha$

By L'Hospital's Rule

$$Z_{IN} = Z_o \frac{[Z_L/\tan\beta S + j\,Z_O]}{[Z_O/\tan\beta S + j\,Z_L]} = j\,Z_O^2 / jZ_L$$

$$\boxed{Z_{IN} = \frac{Z_O^{\,2}}{Z_L}}$$

CASE II: $S = \lambda/2$ (half wavelength)

$$Z_{IN} = Z_o \frac{\{Z_L + j\,Z_O \tan[(2\pi/\lambda)(\lambda/2)]\}}{\{Z_O + j\,Z_L \tan[(2\pi/\lambda)(\lambda/2)]\}}$$

Since $\tan 180^o = 0$

$$Z_{IN} = Z_O \frac{Z_L}{Z_O}$$

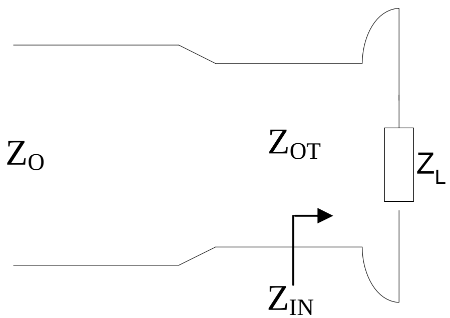
$$\boxed{Z_{IN} = Z_L}$$

MATCHING NETWORKS

<u>General rule:</u> to tune out unwanted load reactance (if any) and the transformation of the resulting impedance to the value required.

I. QUARTER-WAVE TRANFORMER

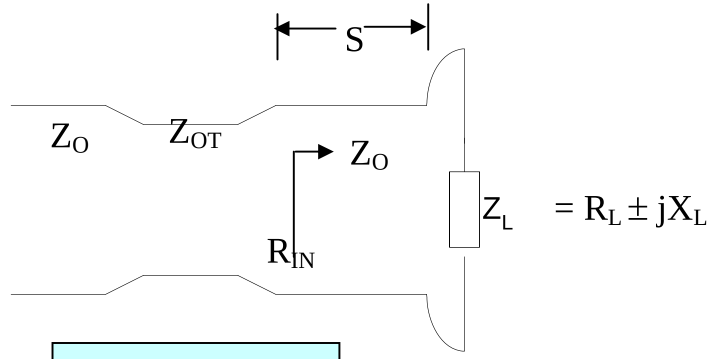
1. $Z_L = R_L$; $R_L \neq Z_O$



$$Z_{IN\lambda/4} = \frac{Z_{OT}^{\;2}}{Z_L}$$

$$Z_{OT} = \sqrt{Z_O R_L}$$

2. $Z_L = R_L \pm jX_L$ (complex) ; $R_L \neq Z_O$



$$Z_{OT} = \sqrt{Z_O R_L}$$

$$S_{MIN} = \frac{\phi + m180^O}{2\beta}$$

$$R_{MIN} = \frac{Z_O}{SWR}$$

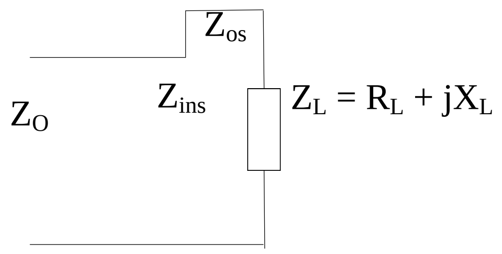
$$S_{MAX} = \frac{\phi + n180^O}{2\beta}$$

$$R_{MAX} = Z_O SWR$$

II. MATCHING STUBS

$Z_L = R_L \pm jX_L$; $R_L = Z_O$

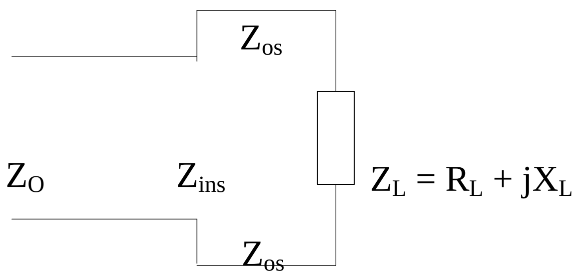
1. Series Short Circuit Stubs



$Z_{ins} = j \tan \beta S (Z_{os}) = -jX_L$

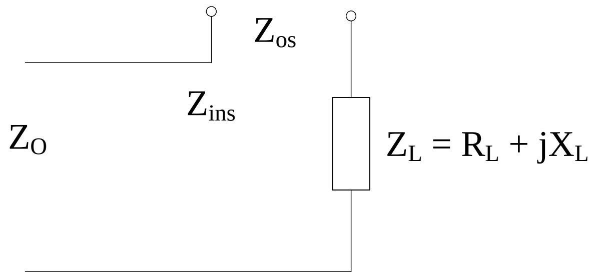
$\beta S = \tan^{-1} (-X_L / Z_{os})$

$Z_{ins} = -jX_L / 2 = j \tan \beta S (Z_{os})$



$\beta S = \tan^{-1} (-X_L / 2Z_{os})$

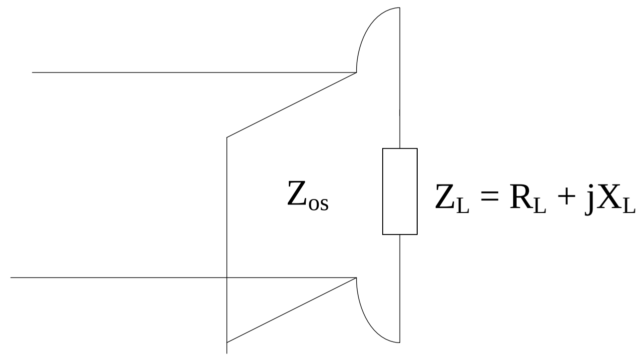
2. Series open Circuit Stubs



$Z_{ins} = -jX_L / 2 = -j \cot \beta S (Z_{os})$

$\beta S = \tan^{-1} (Z_{os} / X_L)$

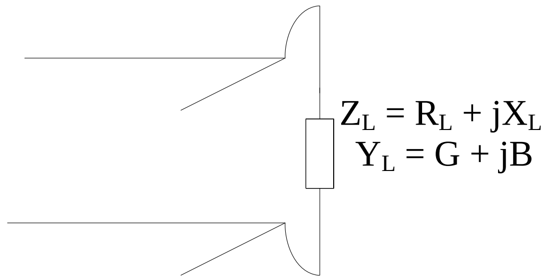
3. Shunt Short Circuit Stubs



$Y_{ins} = 1 / Z_{ins} = -jB = 1 / (j Z_{OS} \tan \beta S)$

$\beta S = \tan^{-1} (Y_{OS} / B)$

4. Shunt Open Circuit Stubs



$Y_{ins} = 1 / Z_{ins} = (j \tan \beta S) / Z_{OS} = -jB$

$\beta S = \tan^{-1} (-BZ_{OS})$ or $\beta S = \tan^{-1} (-B / Y_{OS})$

REACTANCE PROPERTIES OF SHORTED AND OPEN TRANSMISSION LINES