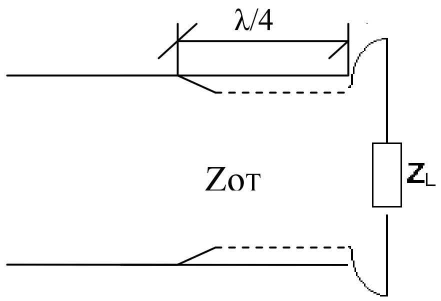
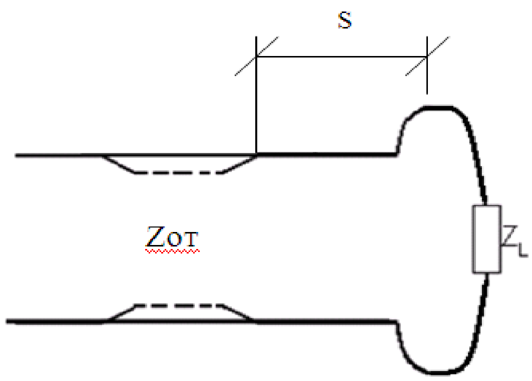
Matching Networks Problems:

1. In order to match a 300-Ω load to a 75- Ω transmission line, a quarter-wave transformer is used. Find the characteristic impedance of λ/4 x'former.
   Solution:



$$Z_O = \frac{{Z_{OT}}^2}{R_L}; Z_{OT} = \sqrt{Z_O R_L} = \sqrt{75(300)} = 150\Omega$$

2. An antenna has a feedpoint of Z_L = (100 – j80) Ω. The characteristic impedance of this antenna is 75 Ω. Determine (a) the shortest distance from the load which a λ/4 x'former maybe inserted to provide matching; (b) Z_{OT}.
   Solution:



$$\Gamma = \frac{Z_L - Z_O}{Z_L + Z_O} = \frac{100 - j80 - 75}{100 - j80 + 75} = 0.436\angle -48.08°$$

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.436}{1-0.436} = 2.55$$

$$S_{max} = \frac{\phi}{2\beta} = \frac{-48.08°}{2(\frac{360°}{\lambda})} = -0.0667\lambda; \textit{ make } \lambda \textit{ positive}: -0.0667\lambda + 0.5\lambda = 0.433\lambda$$

$$S_{\min} = \frac{\phi + 180°}{2\beta} = \frac{-48.08° + 180°}{2(\frac{360°}{\lambda})} = 0.183\lambda$$

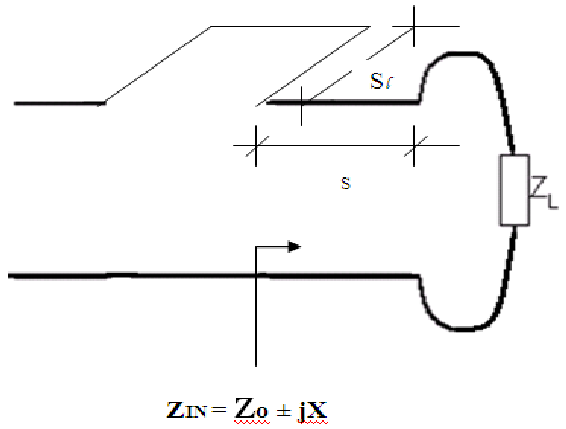
Since S_{min} is closer to the load, choose $S_{min} = 0.183\lambda$.

(b) Since load is resistive: $R_{MIN} = \frac{Z_O}{SWR} = \frac{75}{2.55} = 29.41\Omega$

$$Z_{OT} = \sqrt{Z_O R_L} = \sqrt{75(29.41)} = 46.97\,\Omega$$

3. For a TL with Z_O = 50 Ω and a load impedance of Z_L = (120 + j100) Ω, determine the distance where a series stub must be placed from the load to the line and determine the length of the stub.

Solution:



$$Z_{in} = Z_O \pm jX = 50 \pm jX$$

$$Z_{in} = Z_O \frac{Z_L + jZ_O \tan\beta S}{Z_O + jZ_L \tan\beta S}; let\ \ A = \tan\beta S$$

$$50 \pm jX = 50 \frac{120 + j100 + j50A}{50 + j(120 + j100)A}$$

$$50 \pm jX = 50\frac{120 + j100 + j50A}{50 + j(120 + j100)A}$$

$$50 \pm jX = 50\frac{120 + j(100 + 50A)}{(50 - 100A) + j120A} \bullet \frac{(50 - 100A) - j120A}{(50 - 100A) - j120A}$$

$$50 \pm jX = 50\frac{120(50 - 100A) + (100 + 50A)(120A) + j(100 + 50A)(50 - 100A) - j120(120A)}{(50 - 100A)^2 + 120^2 A^2}$$

Equate real parts:

$$50 = 50\frac{6000 - 12000A + 12000A + 6000A^2}{2500 - 10000A + 10000A^2 + 14400A^2}$$

$$2500 - 10000A + 10000A^2 + 14400A^2 = 6000 - 12000A + 12000A + 6000A^2$$

$$18400A^2 - 10000A - 3500 = 0$$

Solve for the roots: A_1= 0.78561; A_2= -0.24213

$$0.78561 = \tan(\frac{360°}{\lambda})S_1 \;\; ;S_1 \;\; = 0.106\lambda$$

$$-0.24213 = \tan(\frac{360°}{\lambda})S_2 \;\; ;S_2 \;\; = -0.0378\lambda \;\; or \; -0.0378\lambda + 0.5\lambda = 0.4622\lambda$$

Since S_1 is closer to the load, insert the stub at a distance 0.106λ from the load.

$$Z_{in} = 50\frac{120 + j100 + j50(0.78561)}{50 + j(120 + j100)(0.78561)} = 50 - j78.79$$

The series stub must have an input impedance of Z_{ins} = j78.79Ω to tune out the –j78.79Ω reactance.

For the length of the stub:

a) Short circuit stub

$$Z_{ins} = jZ_O \tan\beta S_l; \; j78.79 = j50\tan\beta S_l$$

$$\tan\beta S_l = 1.5758; \; (\frac{360°}{\lambda})S_l = 57.6°$$

$$S_l = 0.16\lambda$$

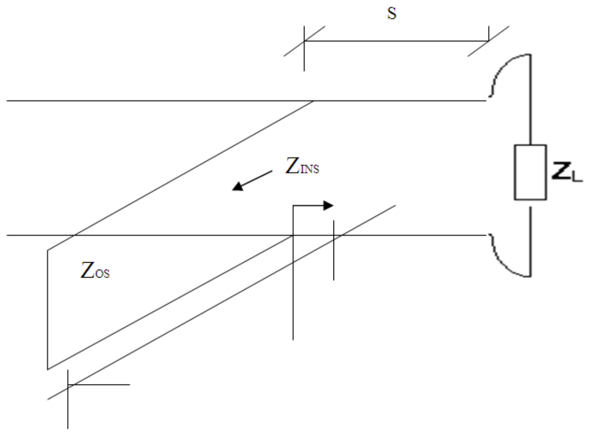
b) Open circuit stub

$$Z_{ins} = -jZ_O \cot \beta S_l;\ j78.79 = -j50 / \tan \beta S_l$$

$$\tan \beta S_l = -0.6346;\ (\frac{360°}{\lambda})S_l = -32.4°$$

$$S_l = -0.09\lambda; making\ \lambda\ \ positive: 0.41\lambda$$

4. A series RC combination having Z_L = (120 - j100) Ω at 10 MHz is connected to a 75 Ω transmission line. Calculate the position and the length of a shunt stub that will match the load to the line.



$$Y_{in} = G \pm jB = 1/Z_{in}$$

$$Y_{in} = \frac{1}{Z_O \dfrac{Z_L + jZ_O \tan \beta S}{Z_O + jZ_L \tan \beta S}}; let\ \ A = \tan \beta S$$

$$\frac{1}{75} \pm jB = (\frac{1}{75})\frac{Z_O + jZ_L A}{Z_L + jZ_O A}$$

$$\frac{1}{75} \pm jB = (\frac{1}{75})\frac{75 + j(120 - j100)A}{(120 - j100) + j75A}$$

$$\frac{1}{75} \pm jB = (\frac{1}{75})\frac{(75 + 100A) + j120A}{120 + j(75A - 100)} \bullet \frac{120 - j(75A - 100)}{120 - j(75A - 100)}$$

$$\frac{1}{75} \pm jB = (\frac{1}{75})\frac{(75 + 100A)(120) + 120A(75A - 100) + j(120A)(120) - j(75 + 100A)(75A - 100)}{120^2 + (75A - 100)^2}$$

Equate real terms:

$$\frac{1}{75}=(\frac{1}{75})\frac{(75+100A)(120)+120A(75A-100)}{120^2+(75A-100)^2}$$

$$120^2+(75A-100)^2=(75+100A)(120)+120A(75A-100)$$

$$144000+5625A^2-15000A+10000=9000+12000A+9000A^2-12000A$$

$$3375A^2+15000A-15400=0$$

Solve for the roots: A_1=0.8602; A_2= -5.305

$$0.8602=\tan(\frac{360°}{\lambda})S_1\ ;S_1\ =0.1131\lambda$$

$$-5.305=\tan(\frac{360°}{\lambda})S_2\ ;S_2\ =-0.2203\lambda\ \ or\ -0.2203\lambda+0.5\lambda=0.2797\lambda$$

Since S_1 is closer to the load, insert the stub at a distance 0.1131λ from the load.

$$Y_{in}=(\frac{1}{75})\frac{75+j(120-j100)(0.8602)}{(120-j100)+j75(0.8602)}=(0.0133336+j0.01541)S$$

The shunt stub must have an input admittance of Y_{ins} = - j0.01541S to tune out the j0.01541 S susceptance.

For the length of the stub:

a) Short circuit stub:

$$Y_{ins}=\frac{1}{Z_{ins}}=\frac{1}{jZ_O\tan\beta S_l};-j0.01541=\frac{1}{j75\tan\beta S_l}$$

$$\tan\beta S_l=0.86524;\ (\frac{360°}{\lambda})S_l=40.8676°$$

$$S_l=0.1135\lambda$$

b) Open circuit stub

$$Y_{ins}=\frac{1}{Z_{ins}}=\frac{1}{-jZ_O\cot\beta S_l};\ -j0.01541=\frac{\tan\beta S_l}{-j75}$$

$$\tan\beta S_l=-1.15575;\ (\frac{360°}{\lambda})S_l=-49.1324°$$

$$S_l=-0.1365\lambda+0.5\lambda=0.3635\lambda$$